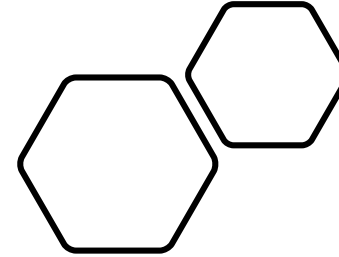


# 15. Boycott Effect



If particles are suspended in a liquid that has a lower density than the particles, the particles will settle to the bottom of the container.

The rate of settling can be affected by tilting the container that holds the liquid.

Explain this phenomenon and investigate the effect of relevant parameters.



Co-funded by the  
Erasmus+ Programme  
of the European Union

Tereza Zuskinová, FMFI UK

# The effect



<https://www.youtube.com/watch?v=8zjixDxTEN8>

# Origins of the effect (1920)

Original article: <https://www.nature.com/articles/104532bo>

## Sedimentation of Blood Corpuscles.

I HAVE noticed lately that if oxalated or defibrinated blood is put to stand in narrow tubes, the corpuscles sediment a good deal faster if the tube is inclined than when it is vertical. Thus with tubes about 2.7 mm. internal diameter there were, after 20 hours, 4, 23, 35, and 42 per cent. of clear serum with tubes inclined at  $0^\circ$ ,  $22\frac{1}{2}^\circ$ ,  $45^\circ$ , and  $67\frac{1}{2}^\circ$  respectively. In another rough experiment with tubes of different diameters, all filled to a height of 40 mm. with diluted blood, after 5 hours there were the following proportions of clear serum:—

mm. diam.	Vertical Per cent.	$11\frac{1}{4}^\circ$ Per cent.	$22\frac{1}{2}^\circ$ Per cent.	$33\frac{3}{4}^\circ$ Per cent.
2.7 ... ..	6	20	29	51
8 ... ..	5	10	15	21
14 ... ..	4	5	9	12

The phenomenon seems to depend on the *vertical* height of the columns of blood, and it occurs to me that the slight Brownian movement of the lower corpuscles may interfere with the sedimentation of those above. But I should be glad if someone would tell me the explanation: the phenomenon is perhaps well known in some other form. A. E. Boycott.

Medical School, University College Hospital, W.C.

English medical scientist  
A. Boycott was studying  
the sedimentation of  
erythrocytes

Origin

Nevskii, Y. A., & Osiptsov, A. N. (2011). *Slow gravitational convection of disperse systems in domains with inclined boundaries. Fluid Dynamics, 46(2), 225–239.*

(20)

Sedimentation of Blood Corpuscles.

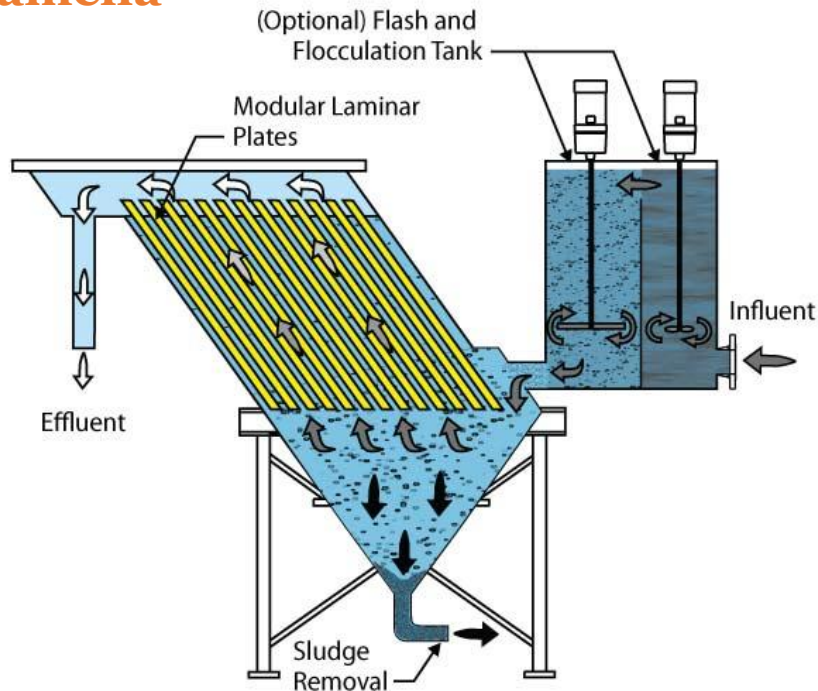
The origins of the development of *vortex zones*, the *division in the particle concentration* field, and the influence of these factors on the effective admixture settling velocity in closed vessels are still not completely understood...

The phenomenon seems to depend on the *vertical* height of the columns of blood, and it occurs to me that the slight Brownian movement of the lower corpuscles may interfere with the sedimentation of those above. But I should be glad if someone would tell me the explanation: the phenomenon is perhaps well known in some other form. A. E. Boycott.  
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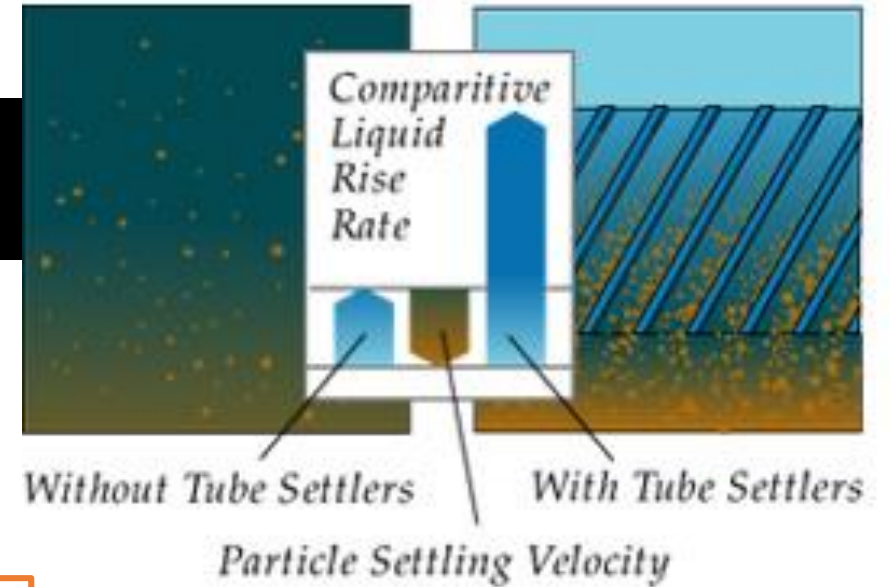
# Motivation

## Sedimentation for purifying water

### Lamella

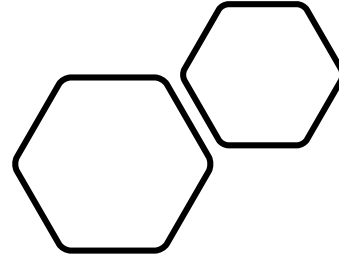


### Tube Settlers vs. Conventional Settling

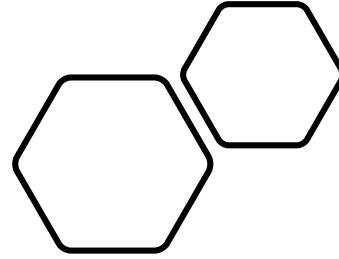


Creates a larger surface where the sediment can settle

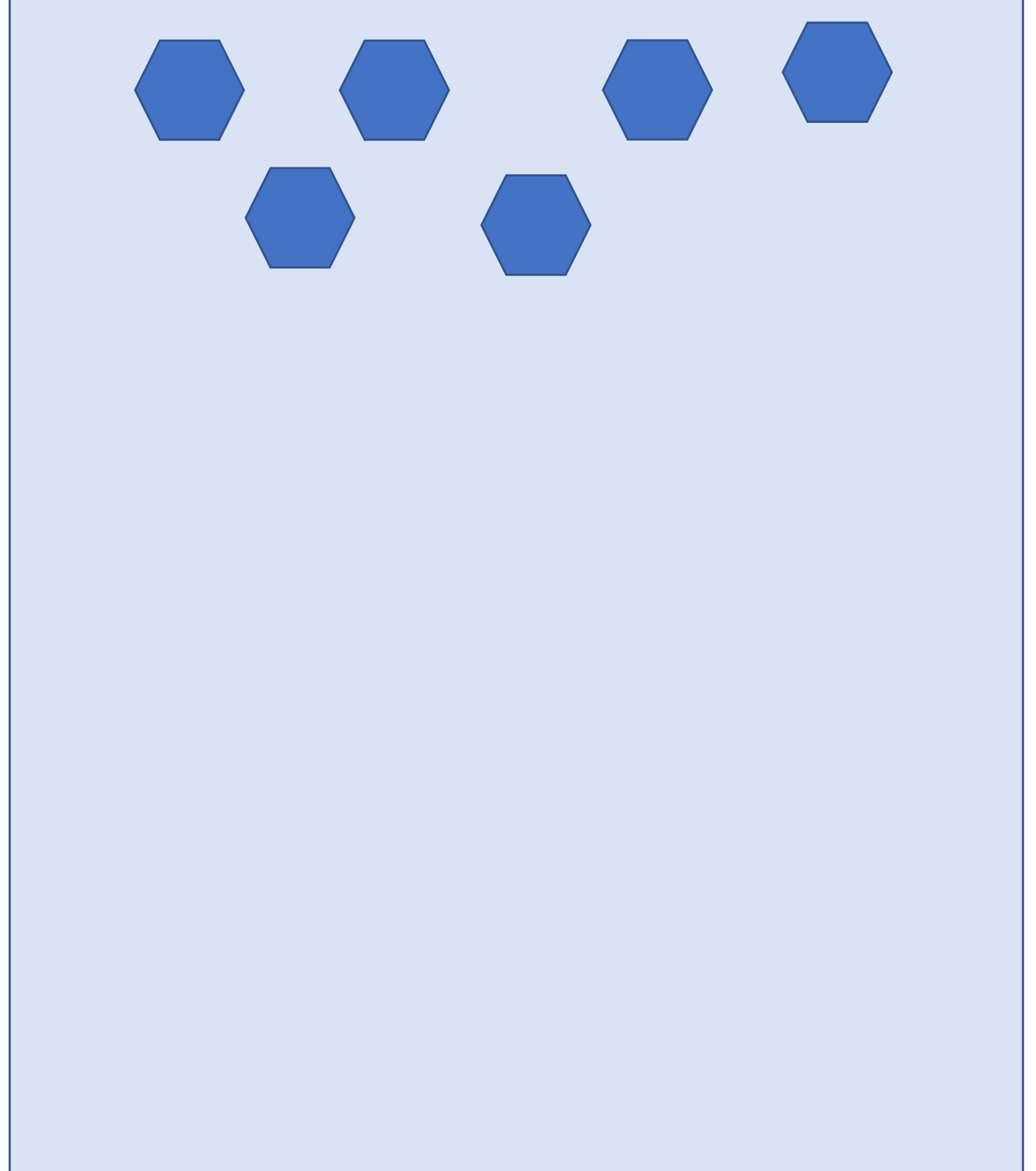
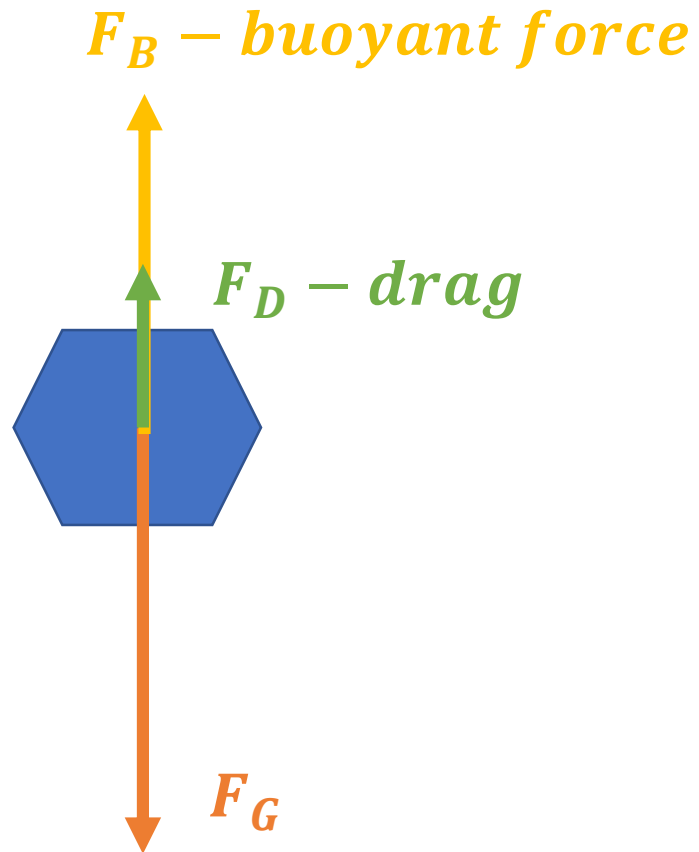
# Basic explanation



Without tilt

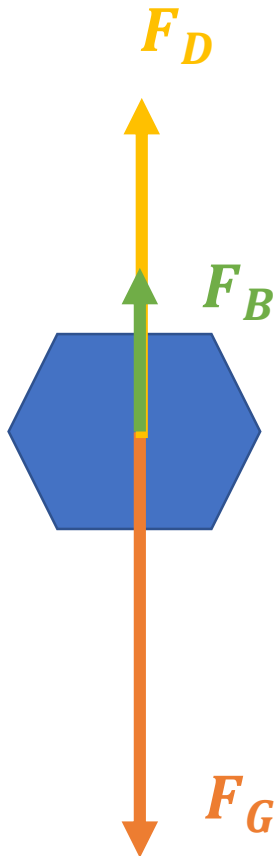


# Force analysis



In order to start to fall  $F_G > F_B + F_D$





Gravitational force:

$$F_G = mg = \rho V g$$

*Relevant parameter: MATERIAL -> DENSITY*

*Relevant parameter: VOLUME*

$$\rho_{sand} \approx 1.442 \text{ g/cm}^3 > \rho_{water} = 1 \text{ g/cm}^3$$

*If particles are suspended in a liquid that has a lower density than the particles, the particles will settle to the bottom of the container.*

*The rate of settling can be affected by tilting the container that holds the liquid.*

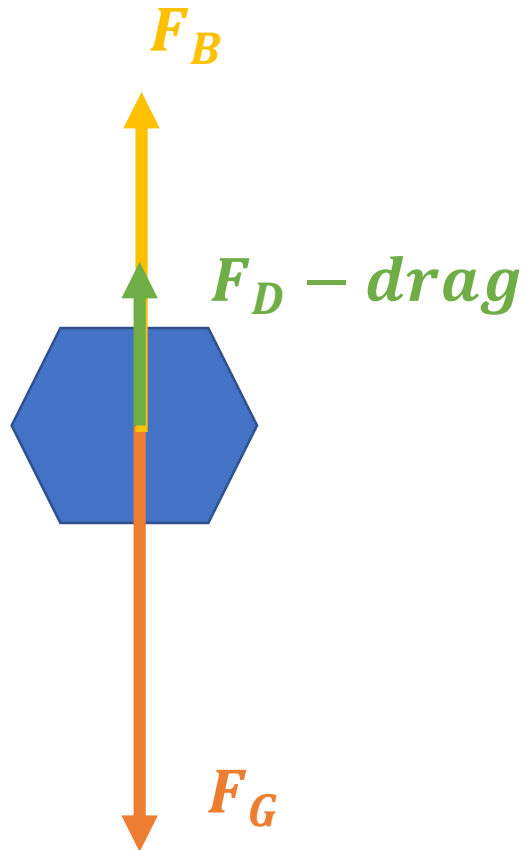
*Explain this phenomenon and investigate the effect of relevant parameters.*

# Materials

What to use?

- Sand, coffee grounds, glitter, microbeads- something of a diameter  $d \ll D_{tube}$  or explore the ratio of these two
- Good not to dissolve to see the effect and reproduce data with same particles
- Articles suggest that we should use **concentration of particles** 0.05-0.4

# Force analysis



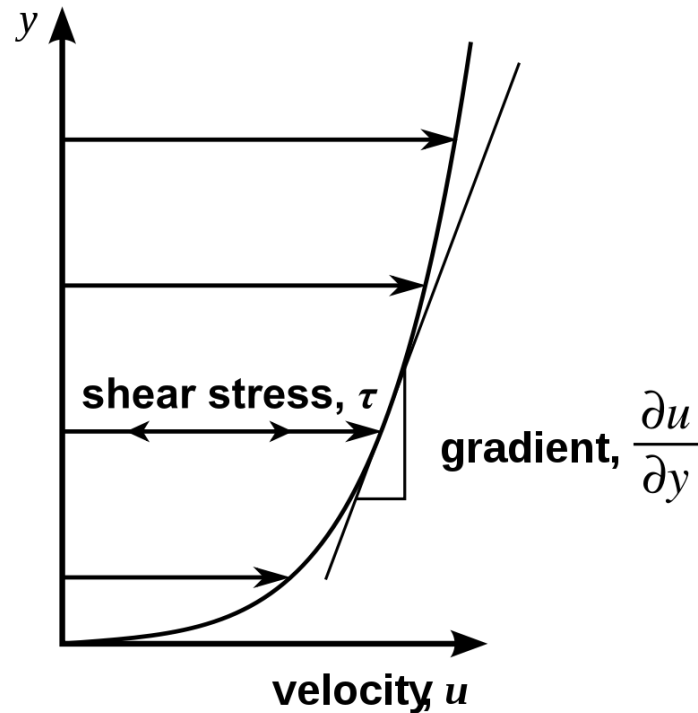
Drag force for small speeds, no turbulence (i.e. low [Reynolds number](#)  $R_e < 1$ ), Stokes force can be used :

$$F_D = -6\pi\eta r v$$

$\eta$  is viscosity of fluid,  $r$  radius of a sphere,  $v$  speed of a piece.

*Relevant parameter: FLUID VISCOSITY*  
*Relevant parameter: PARTICLE RADIUS*

# Viscosity as a key parameter



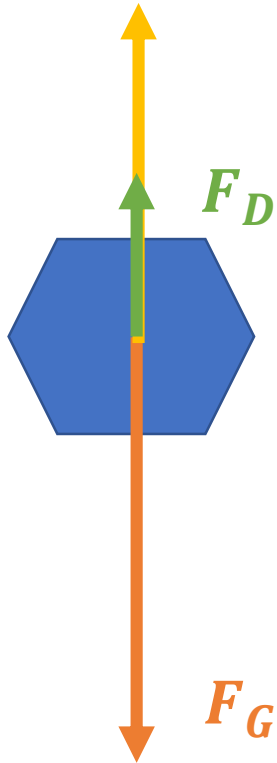
$$\tau = \mu \frac{\partial u}{\partial y}$$

$\mu$  is the dynamic viscosity of a liquid

**Change the liquids (see-though)**  
**Water (1.0016 Pa.s),**  
**Oils (0.8-3 Pa.s),**  
**Glycerol (1.412 Pa.s)**

# Force analysis

$F_B$  – Buoyant force



Buoyancy - Archimedes force

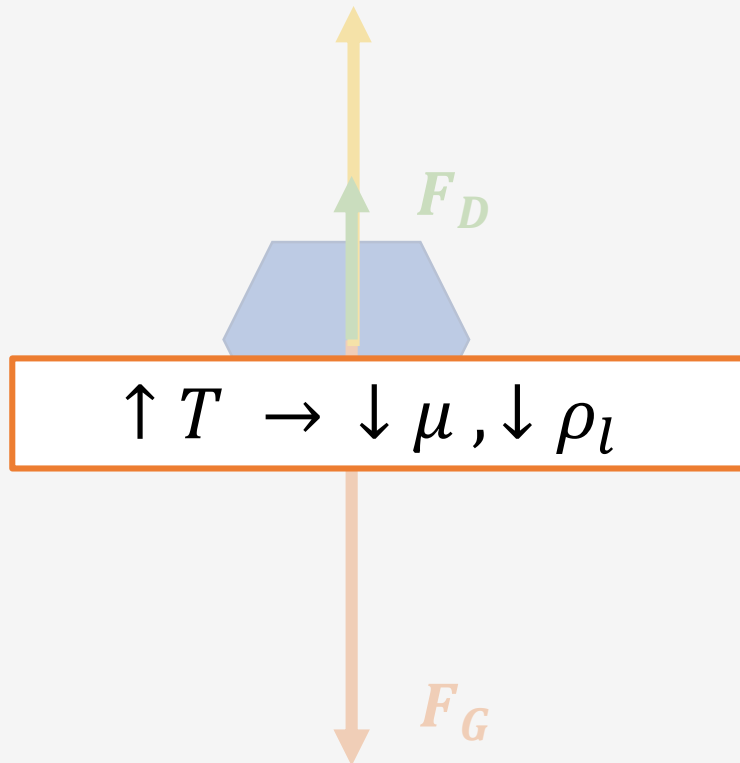
$$F_B = -\rho_l V g$$

$\rho_l$  is the liquid density,  $V$  Volume of a particle.

*RELEVANT PARAMETER: Density of the liquid*

# Force analysis

$F_B$  – Buoyant force

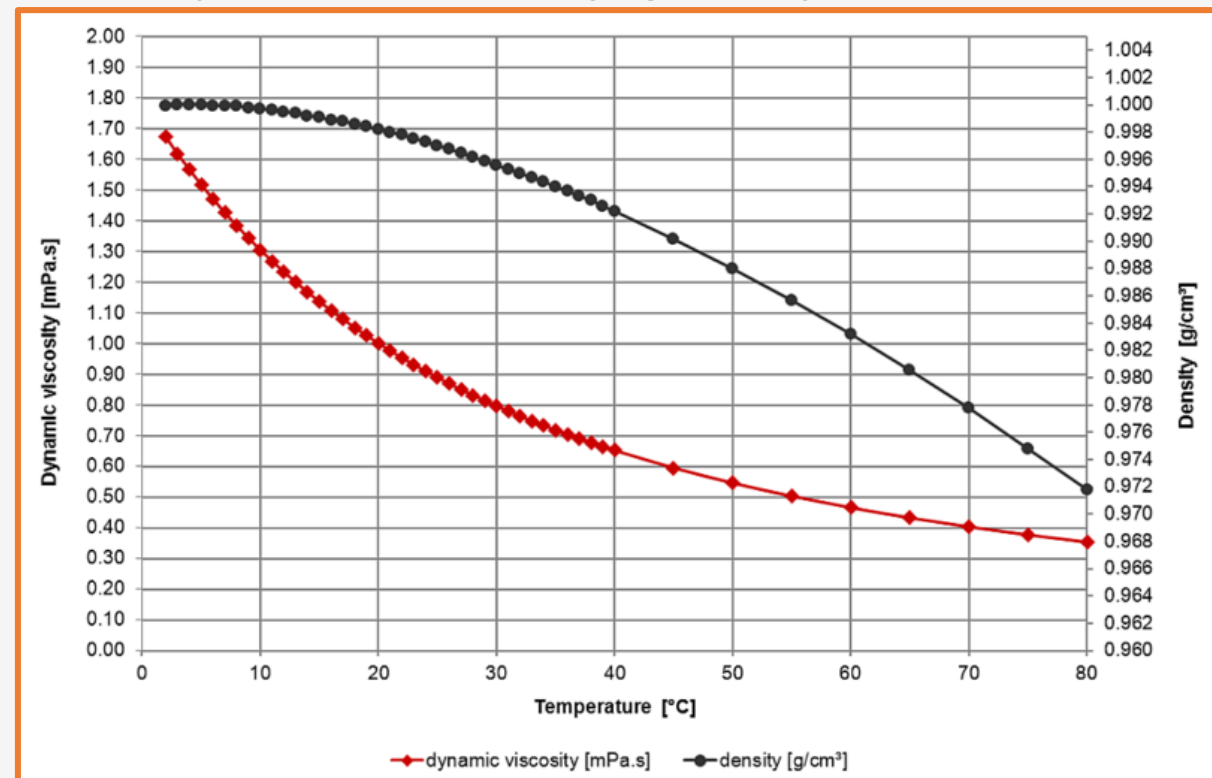


Buoyancy - Archimedes force

$$F_B = -\rho_l V g$$

$\rho_l$  is the liquid density,  $V$  Volume of a particle.

Relevant parameter: Density of the liquid



Differential equation

$$m\mathbf{a} = \mathbf{F}_G - \mathbf{F}_B - \mathbf{F}_D$$

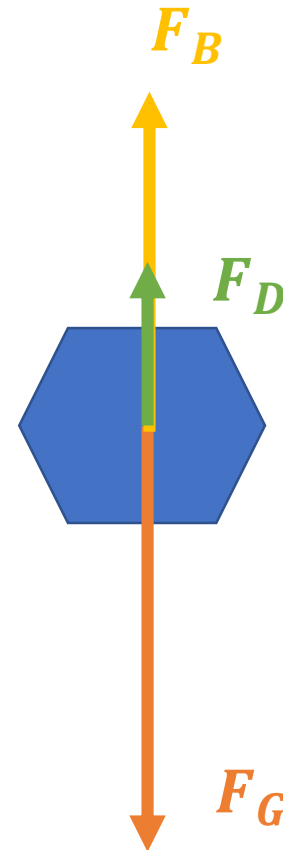
$$m \frac{dv}{dt} = mg - \rho_l g V - bv$$

$$b = 6\pi\eta r$$

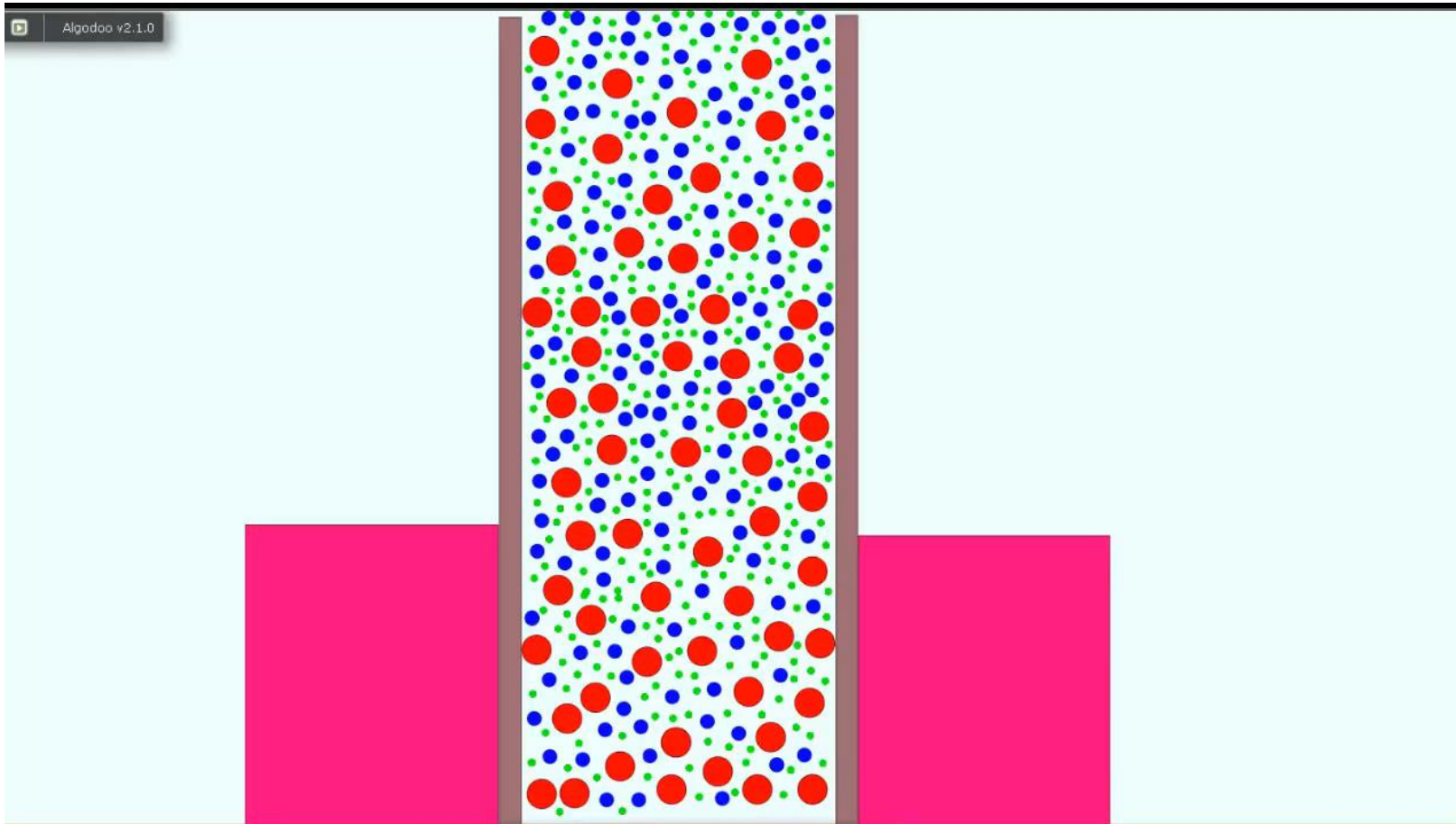
$$v(t) = \frac{(\rho - \rho_l)Vg}{b} \left( 1 - e^{-\frac{bt}{m}} \right)$$

$$v_{terminal} \rightarrow const = \frac{(\rho - \rho_0)Vg}{6\pi\eta r}$$

As we don't have to have spheres – need to experimentally verify  $v(t)$  by tracking the particles



# Visualisation for different radii



As the speed of the particles depends on their radius, the biggest particles are the one with the highest speed.

$$v_t = \frac{(\rho - \rho_0)Vg}{6\pi\eta r}$$

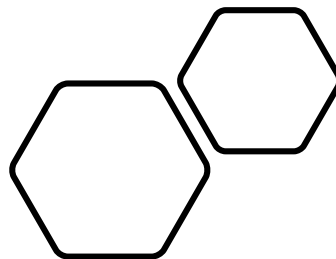
$\uparrow r, \uparrow v_t$

*Nice to play with* - Software **Algodoo**, where you can add viscosity and different shapes or particles.

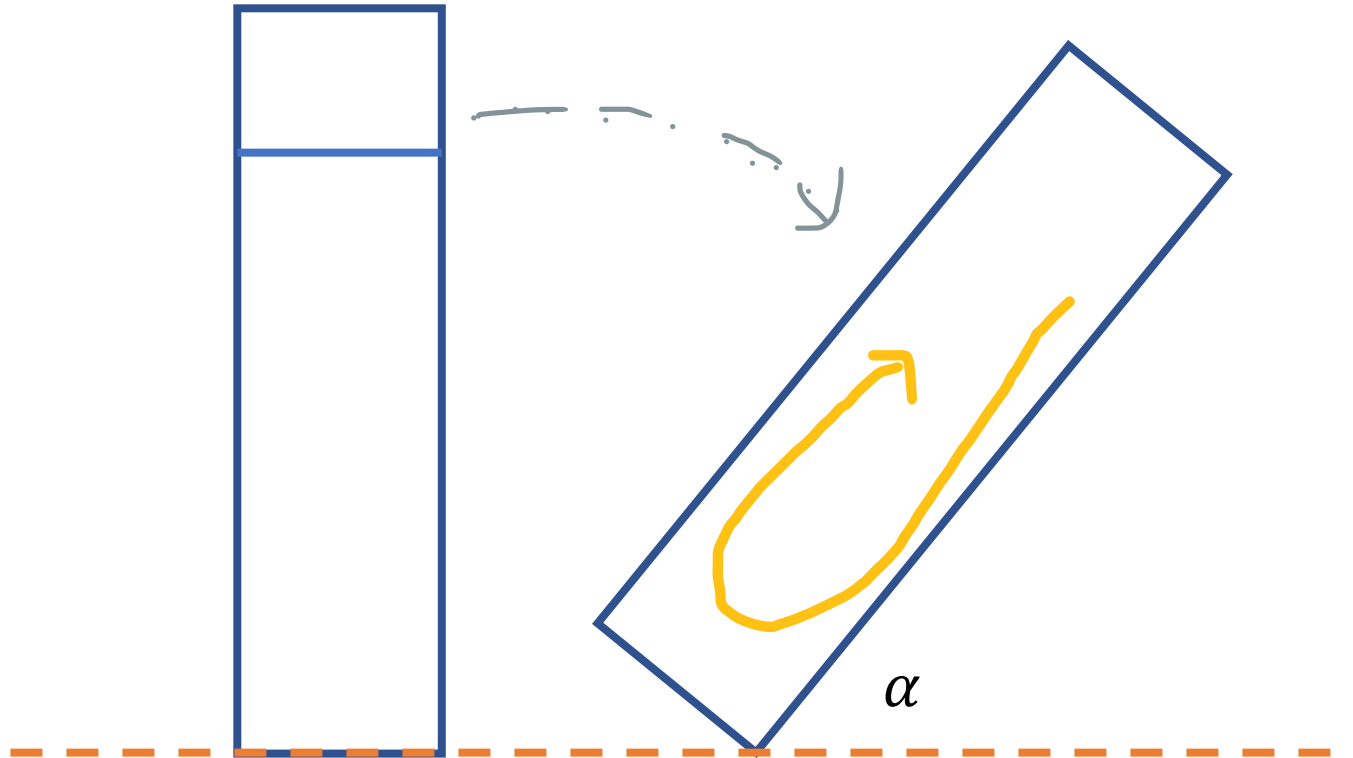
<https://www.youtube.com/watch?v=9XFTqcw1tX0>



With tilt



# Fluid currents



Now we add  
the particles

# Convection created by particles

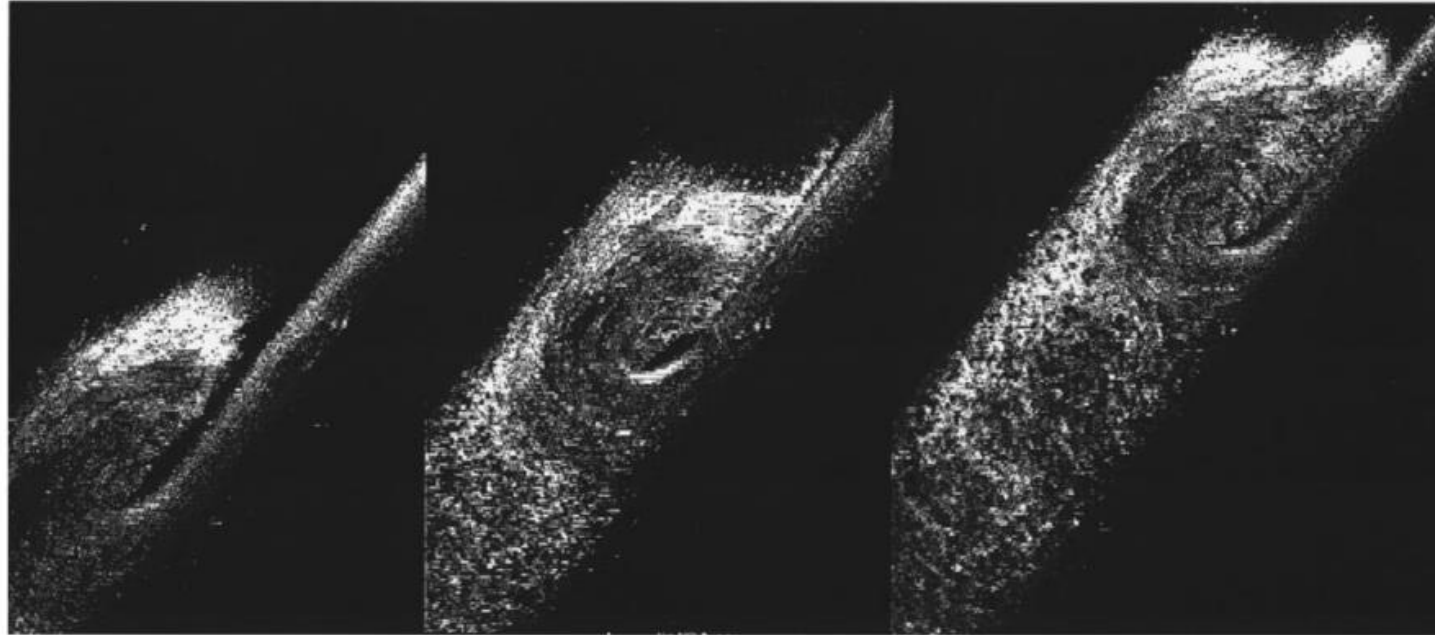
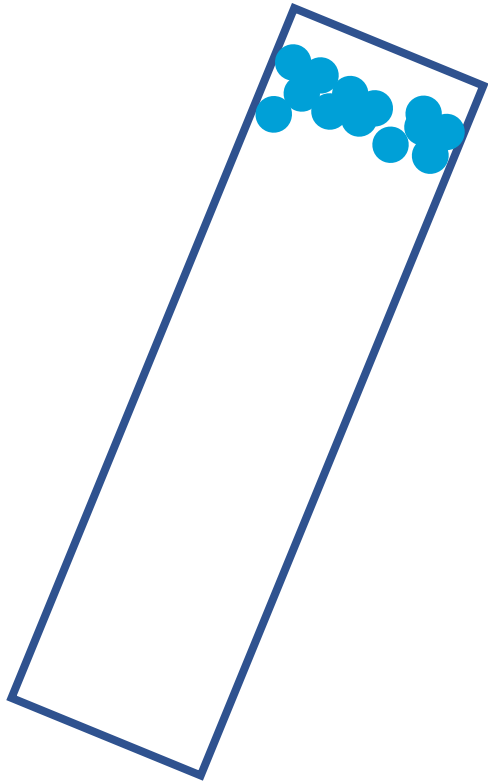


FIG. 2. A sequence of snapshots at 20-ms intervals of the convective roll in an inclined tube of granulate. The flow is from right to left.

*Maybe recreate this to visualise the movement of the water with some light fluorescent granulate*

The particle's velocity relative to the fluid is still the same

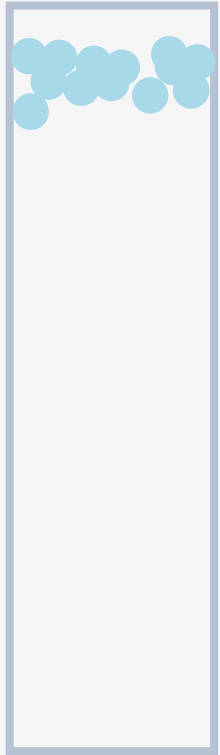
$$v_{terminal} = \frac{(\rho - \rho_0)Vg}{6\pi\eta r}$$



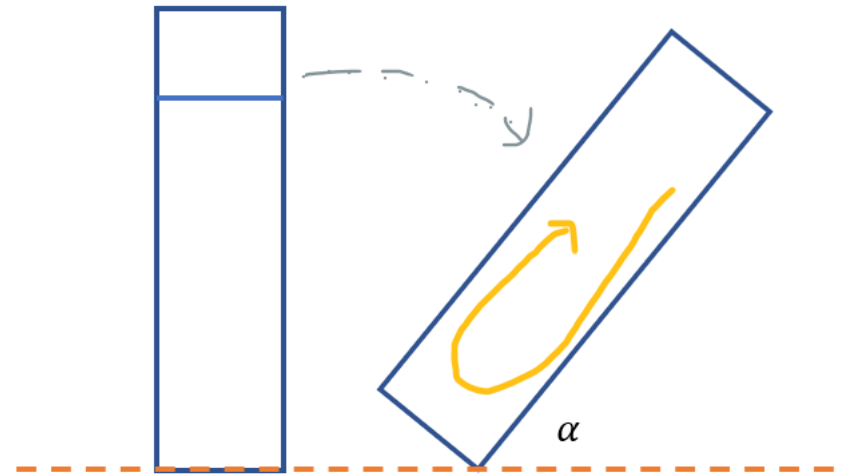
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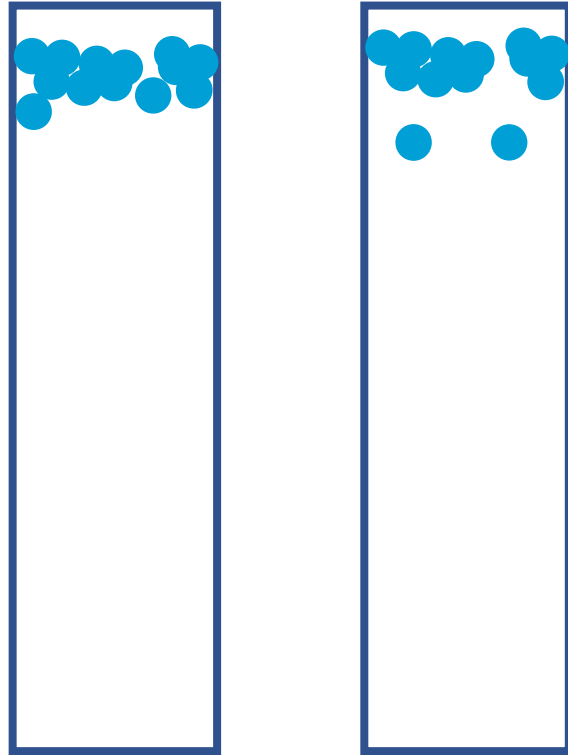
$$v_{terminal} = \frac{(\rho - \rho_0)Vg}{6\pi\eta r}$$

It is the current of the flow that speeds up the particles



Fluid currents





The particle's velocity relative to the fluid is still the same

$$v_{terminal} = \frac{(\rho - \rho_0)Vg}{6\pi\eta r}$$

It is the current of the flow that speeds up the particles

#### Observation

Material does not fall uniformly  
the upper part of the packed material remains relatively stationary, while the lower part falls first.

#### **Key point:**

Particles start to settle down



induce the development of convective motion as they form the “regions”

# Regions

There are effectively at least **three** characteristic regions to account for

- They have different densities thus different hydrostatic and gravitational forces act on them

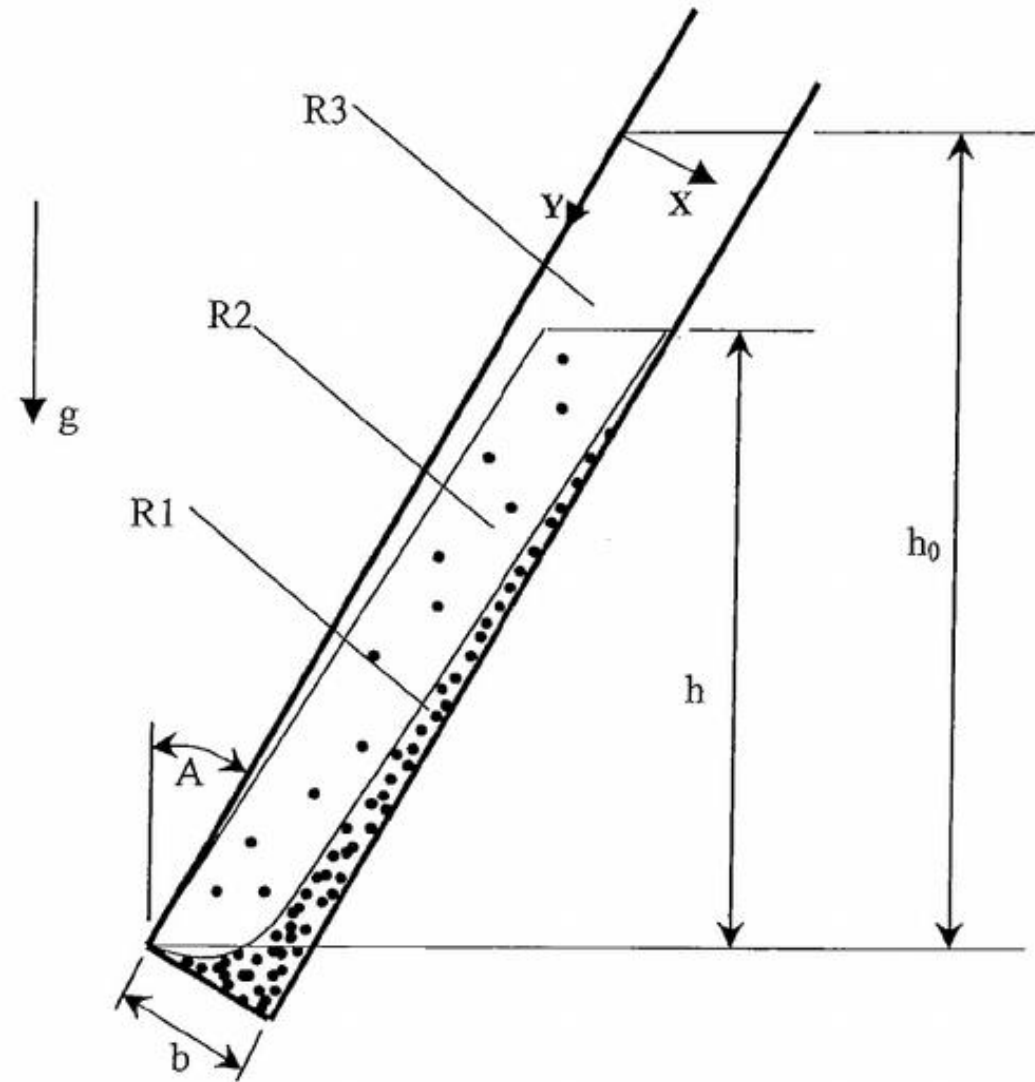
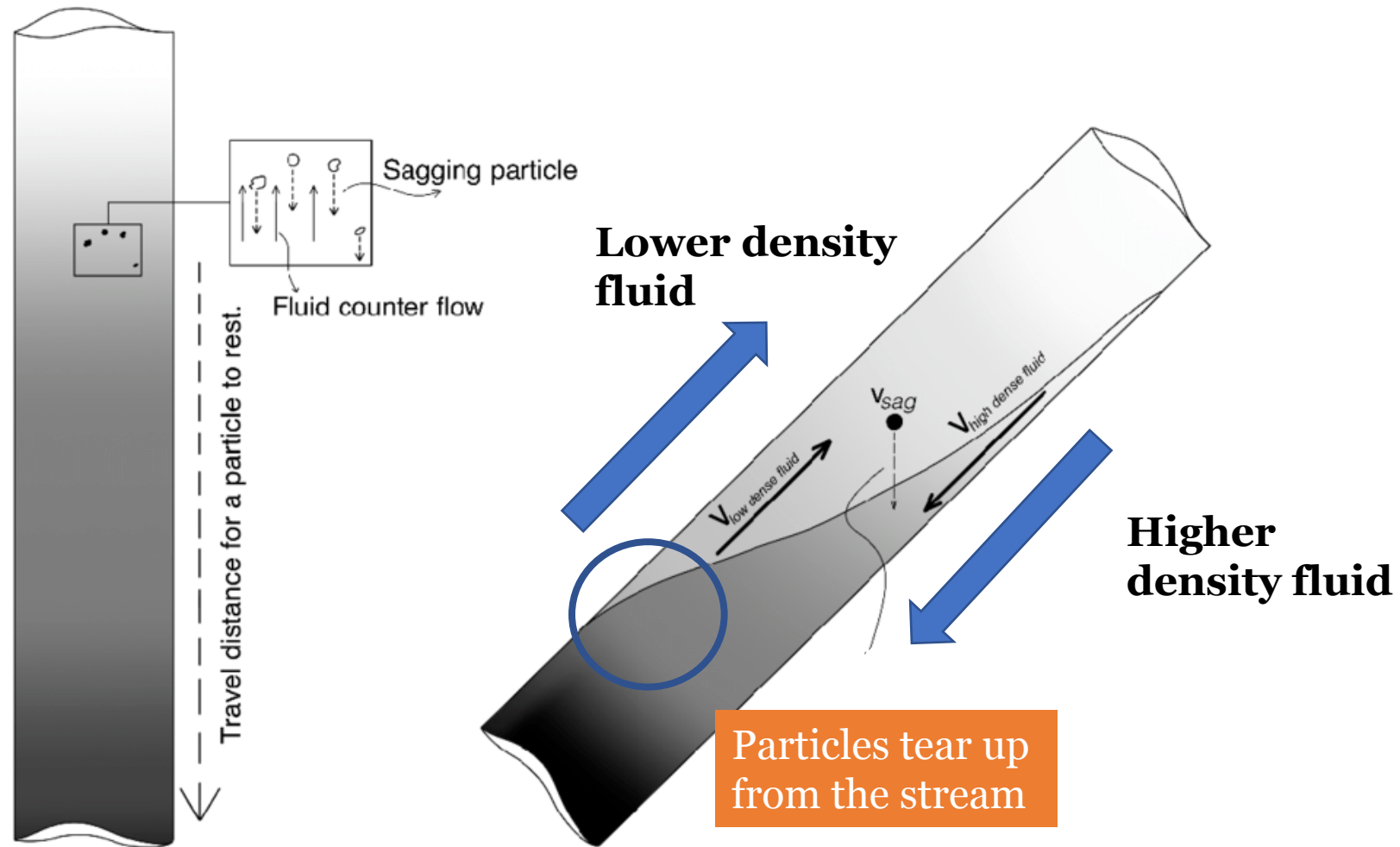


Figure 1. Illustration of the different regions for particles settling in an inclined container.

# Density difference





The particles  
tear form the  
upward stream

Makes the  
upward going  
fluid more  
buoyant



<https://www.youtube.com/watch?v=i1oA8B83180>

# PNK theory (~1925)

$$\frac{dh}{dt} = -v_0 \left( 1 + \frac{h}{b} \sin\theta \right)$$



Can we quantitatively explain this phenomenon?

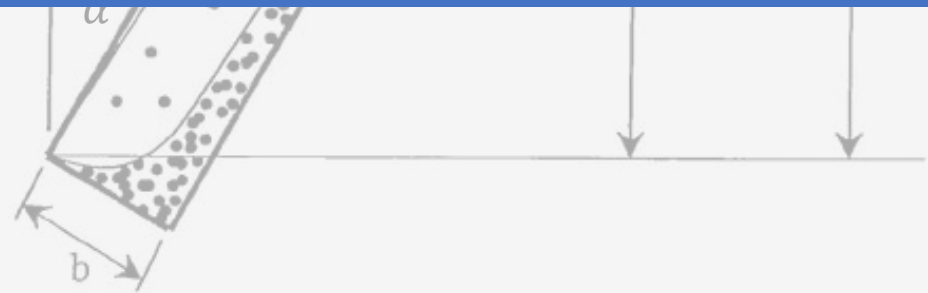


Figure 1. Illustration of the different regions for particles settling in an inclined container.

# PNK theory (~1925)

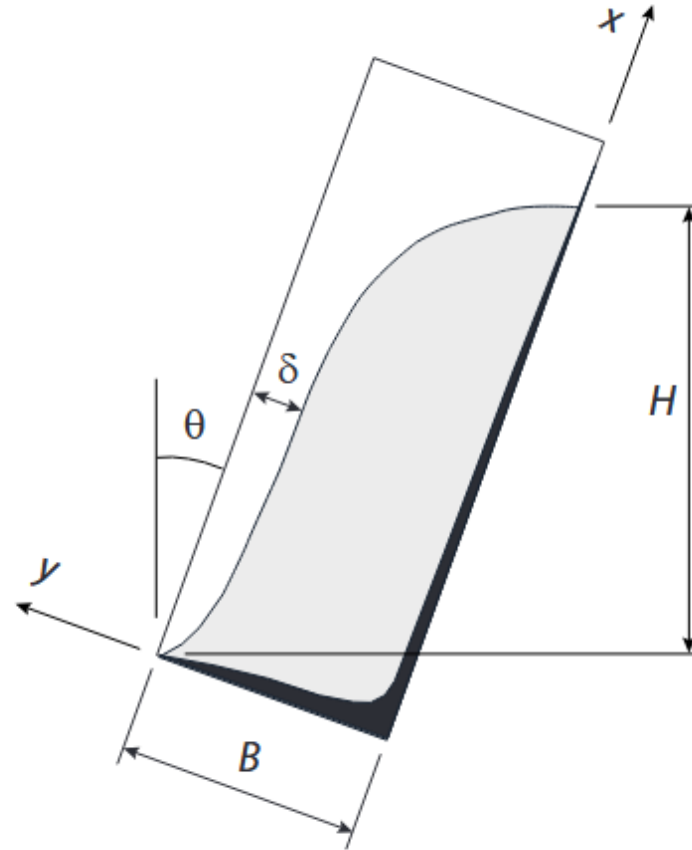
$$\frac{dH}{dt} = -v_0 \left( 1 + \frac{h}{B} \sin\theta \right)$$

Where  $v_0$  is the suspension interface velocity (in R2)

The rate of increased by factor  $\frac{h}{B} \sin\alpha$ .

Futher reading

<https://people.eng.unimelb.edu.au/daltonh/bibliography/downloads/dhispsec03.pdf>



# Dependence on the angle

Acrivos, A., & Herbolzheimer, E. (1979). Enhanced sedimentation in settling tanks with inclined walls. *Journal of Fluid Mechanics*, 92(03), 435.  
doi:10.1017/S0022112079000720

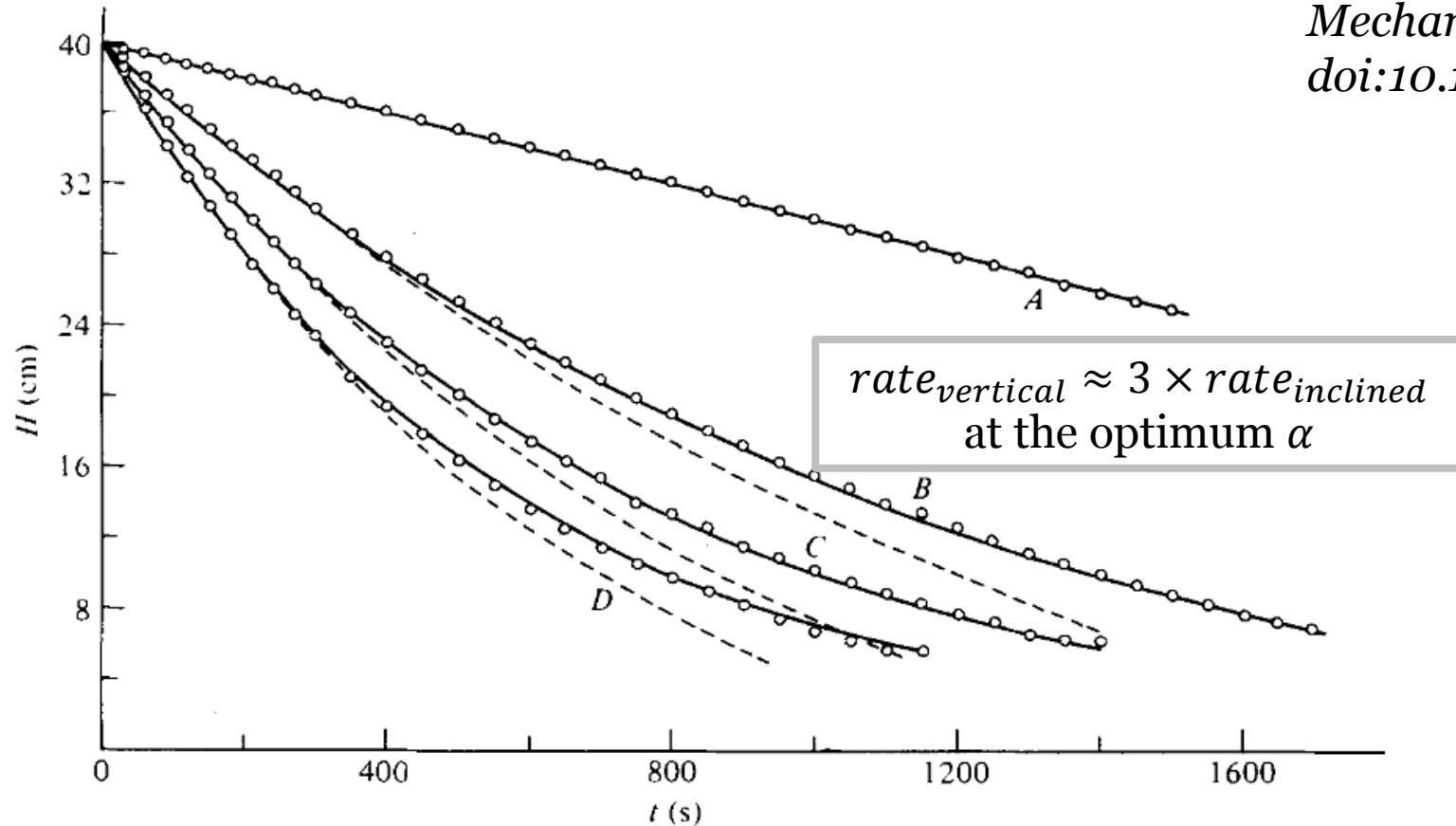
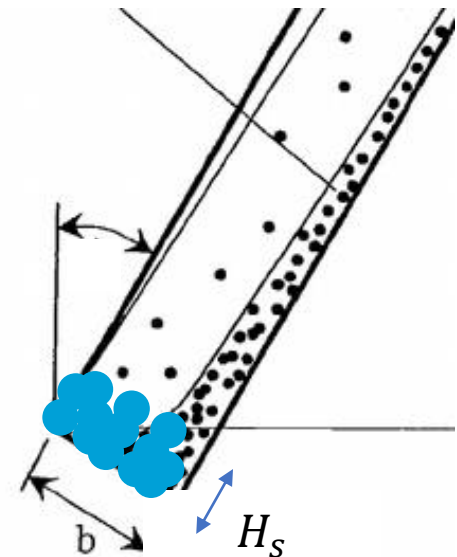


FIGURE 4. Height of the top interface  $H$  versus time for  $c_0 = 0.10$ ,  $H_0 = 40$  cm, and  $b = 5$  cm ( $\Lambda_0 = 3.27 \times 10^7$ ,  $R_0 = 0.560$ ) for different angles of inclination  $\alpha$ : A,  $\alpha = 0^\circ$ ; B,  $\alpha = 20^\circ$ ; C,  $\alpha = 35^\circ$ ; D,  $\alpha = 50^\circ$ . ---, ordinary PNK theory; —, PNK predictions accounting for the sediment layer.

$$\left. \begin{aligned} R &\equiv l\rho_f u_0/\mu = \frac{2}{9}l a^2 \rho_f (\rho_s - \rho_f) g/\mu^2, \\ \Lambda &= l^2 g (\rho_s - \rho_f) c_0 / u_0 \mu = \frac{9}{2} (l/a)^2 c_0, \end{aligned} \right\}$$

Note to keep the particle concentration constant in one experiment



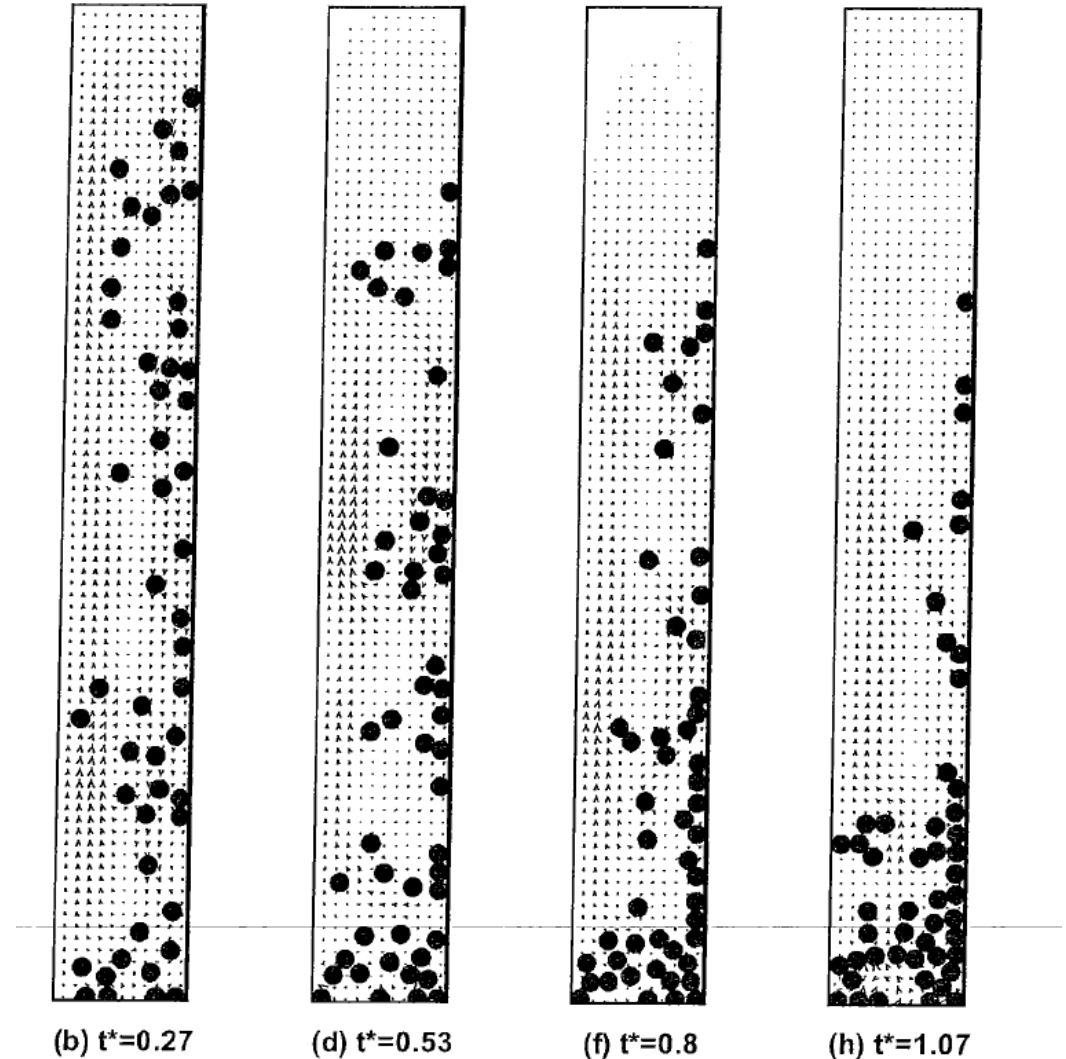
# Further reading (free access)

Numerical Simulations of the BE

<https://www.researchgate.net/publication/232849633> A Numerical Simulation of the Boycott Effect

*Their computations suggest-*For angles with inclination higher than  $45^\circ$  settling rate is slower and global convection becomes weaker – it's due to the weaker driving force (gravitational)

Equilibrium seems to be  $45^\circ$



# Further reading

<https://www.researchgate.net/publication/232005098> The stratified Boycott effect

Palma, Sergio, et al. "Characterization of a Sediment Layer of Concentrated Fluid-Solid Mixtures in Tilted Ducts at Low Reynolds Numbers." Powder Technology, Elsevier, 3 Nov. 2017,

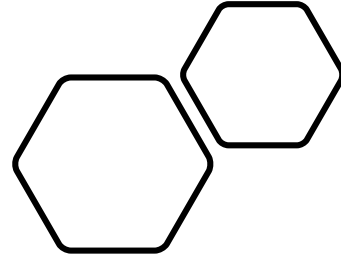
[www.sciencedirect.com/science/article/pii/S0032591017308495](http://www.sciencedirect.com/science/article/pii/S0032591017308495).

Peacock, Tom, et al. "The Stratified Boycott Effect." Journal of Fluid Mechanics, vol. 529, 2005, pp. 33–49., doi:10.1017/S002211200500337x.

The Editors of Encyclopaedia Britannica. "Stokes's Law." Encyclopædia Britannica, Encyclopædia Britannica, Inc., 11 Apr. 2016, [www.britannica.com/science/Stokess-law](http://www.britannica.com/science/Stokess-law).

Baranets, Vitaliia, and Natalya Kizilova. "Mathematical Modeling of Particle Aggregation and Sedimentation in the Inclined Tubes." Visnyk of V. N. Karazin Kharkiv National University. Ser. Mathematics, Applied Mathematics and Mechanics, 2019, [periodicals.karazin.ua/mech\\_math/article/view/14948](http://periodicals.karazin.ua/mech_math/article/view/14948).

To Do



# What to do with the problem?

## Easy level

- Straightforward experiments, changing parameters
- Measuring the speeds from videos, dependence on the angle  $v(\alpha)$ , viscosity, density of the fluid, investigate particle concentration, find the optimum angle.

## More advanced

- Some theoretical model to compare your results with experiments
- Look over some theoretical simulations available in articles (<https://www.researchgate.net/publication/232849633> A Numerical Simulation of the Boycott Effect~ and others...)
- Add things like adhesion and cohesion between the particles

Good luck 😊